FINDING EQUATIONS FOR LINES IN $\mathbf{R}^3$

For a line you need
1. A POINT on the line, which we can write using a position vector $\mathbf{r}_0 = <x_0, y_0, z_0>$.
2. A DIRECTION vector, $\mathbf{v} = <a, b, c>$.

A parametric equation for the line is given by
$$<x, y, z> = <x_0, y_0, z_0> + t<a, b, c>$$
so

$\cdot$ $\boxed{x = x_0 + at, \ \ y = y_0 + bt, \ \ z = z_0 + ct}$

Here is how you find the equations for a line given various scenarios:

I: Are you given two points, $A(x_0, y_0, z_0)$ and $B(x_1, y_1, z_1)$?
You can get the direction vector as
$$\mathbf{v} = \,<a, b, c> \,= \,<x_1 - x_0, y_1 - y_0, z_1 - z_0>$$

II: Can you find two points?
If you can find two points you can use method I. For example, if you are asked to find the equation of the line of intersection of two planes, you can start by find any two points on the intersection then use method I.

III: Are you given the information already?
Does the problem say the desired line is parallel to another given line, then you already have a direction vector (the direction vectors are the same). Does it say it is perpendicular to a given plane, then you already have a direction vector (the direction vector is the normal).

FINDING EQUATIONS FOR PLANES IN $\mathbf{R}^3$

For a plane you need
1. A POINT on the plane, $\mathbf{r}_0 = \ <x_0, y_0, z_0>$.
2. A NORMAL vector $\mathbf{n} = \ <a, b, c>$.

The equation for the plane then is

$$<a, b, c> \cdot <x - x_0, y - y_0, z - z_0> = 0$$

which becomes

$$\boxed{a(x - x_0) + b(y - y_0) + c(z - z_0) = 0.}$$

I: Are you given three points, $A(x_0, y_0, z_0)$ $B(x_1, y_1, z_1)$, and $C(x_2, y_2, z_2)$ on the plane? (Or two vectors parallel to the plane?)

Use the cross product as follows:

Find the vectors (or just use them if they are given).

$\overrightarrow{AB} = \; < x_1 - x_0, \; y_1 - y_0, \; z_1 - z_0 >$ and

$\overrightarrow{AC} = \; < x_2 - x_0, \; y_2 - y_0, \; z_2 - z_0 >$

and compute: $\mathbf{n} = \; < a, b, c > = \overrightarrow{AB} \times \overrightarrow{AC}.$

II: Can you find three points?

If you can find three points you can use method I. For example, if you are asked to find the equation of the plane containing two intersecting lines, you could find any three points on the two lines (not all on same line).

III: Are you given the information already?

Are you told the plane is perpendicular to some other line, then you already have the normal. And so on.